Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

THE TEACHING OF LOCUS PROBLEMS IN ELEMENTARY GEOMETRY.*

Locus problems constitute a part of geometry usually dreaded alike by pupil and teacher. The pupil may be confident of his ability to demonstrate all the propositions and may be master of the facts and processes that are the basis of numerical work, yet the area tenanted by locus problems will appear to him a trackless and endless morass in which are lurking perils innumerable. The teacher is in doubt as to which of the unpromising paths to choose, and, when once plunged into the maze, where he may safely halt and reconnoiter. The travail of such repeated experiences has brought forth the outline below.

Before turning to the outline in detail, a warning should be uttered against taking up locus problems too early. They are not adapted to the beginner. There is a general opinion today that demonstrative geometry may well be preceded by considerable work in constructive or intuitional geometry, and this because rigorous reasoning is not a function of the immature mind. Similarly the idea of the locus is a still more highly generalized conception, and should bide its time. If a course in Plane Geometry is to consist of a beginning year followed by a review course, little if any locus work should be done the first year. If all is to be crowded into one year, the locus element should be held in abeyance until the latter half of the course. Much time and vexation is saved by such delay.

Moreover, a scheme that delays the introduction of the locus idea until the class is approaching the end of its course in geometry affords an opportunity to use it for review, an incidental use to which it is particularly well adapted. The ordinary locus problem involves no new principles or facts. It merely re-groups old facts, and faces them from a new point of view. From a pedagogical standpoint, it occupies about the same place that the topic of inverse functions does in trigonometry. Both consist of old facts under new statements, and both serve alike for effective review. If these topics find a place in a course the moment the material for them is at hand, their function as reviews and summaries is largely lost.

---

* Delivered before the Association of Teachers of Mathematics in New England, December 4, 1920.

The outline below, the outgrowth, as has been said, of actual class-room work, is constructed from the practical rather than from the theoretical view. It seeks to lead the pupil in two respect. First, it shows him how to formulate the knowledge he has on a stated problem in the form and language that the locus conception has by convention assumed; that is, to tell him what he ought to say, in what form he should arrange what he says, and when he has said enough. It aims to give him confidence when he finishes a discussion that he has covered the ground sufficiently, and also to develop in him a check against "running to useless words." Second, the outline will aid the pupil to discover systematically such facts as he may need for the basis of his conclusions. In short, it supplies a definite working rule, a common ground between teacher and pupil.

It may be added, moreover, that the method of treatment of the outline is determined by the limitations of elementary mathematics rather than by any ulterior aim. Courses in college may often, even in the freshman year, find it advisable to shift the emphasis of the point of view. There is no real difficulty in this if the student brings to his new task a clear and fundamental idea of the nature of a locus. The best preparation for advanced loci is a mastery of elementary loci.

The material in larger type is intended to be placed before the pupil. He should become familiar with the heads and subheads. The supplementary explanatory matter in the small type is primarily for the teacher.

OUTLINE FOR HANDLING LOCUS PROBLEMS.

A. Definition.

A locus is the goemetrical figure occupied by a series of points (or lines), that fulfill a given condition.

The pupil's attention may well be directed to the fact that the word locus means in geometry just what it has meant in Latin: namely, the *place* where an object is. The words "geometrical figure" cover all the geometrical elements, line, point, surface, solid. The phrase, "series of points" suggests correlation, and in a way, continuity. The words in parenthesis "or lines" are not likely to be needed at first, not until solid geometry is reached. It seems that the conception of a locus or a figure is better adapted to beginners than that of the path of a moving point.

Many teachers will at once miss in the definition a final qualifying phrase, "and no other points." This is purposely omitted. There is, of course, no thought of suggesting that the locus should not be restricted to its minimum limits. It is believed, however, that the strict meaning of the word "occupied" itself is sufficiently restrictive. Such parts of an assumed locus that are not definitely *occupied* by any of the points in question will be ruled out of the application by a strict interpretation of this verb. There is a marked advantage in having any definition concise, and also in avoiding, as far as may be, the distractions of qualifying phrases. If it be feared that the pupil will not sufficiently assimilate the restrictive element from the wording of the definition, he will certainly receive it from Part III. in the following discussion of loci.

B. Discussion of Simple Loci.

1. The discussion of a locus problem consists of *four stages.* *Discover* and *state* accurately what the locus called for is. This step is unnecessary if the problem as given states itself the locus in question.

(*a*) To *discover* a locus, plot a few points, wisely scattered, that satisfy the given condition, and from these forecast the complete locus.

This process of systematically proceeding to discover a locus is often wholly or largely unnecessary, inasmuch as in many cases it is evident at once or almost at once what the locus in general terms is. This process of discovery has much about it akin to developing the negative of an unknown photograph, in which process the strong features come out gradually until soon the subject of the plate is evident. That which is meant by the phrase "wisely scattered" hardly needs explanation. The pupil should be taught to place some of the points he is using toward the extreme positions, and especially in peculiar ones. The word "forecast" suggests the proper attitude of the pupil's mind during this process.

(*b*) To *state* what a locus is, give attention to (1) what it is, (2) where it is, (3) how big it is. As to (1), it may be a point, a line, a surface, or even a solid, but in plane geometry is usually a line, straight or curved.

This power of statement is really the heart of any locus problem, and cannot be too much emphasized. If the pupil considers in turn, in forming his statement, the three points just mentioned, he is not likely to omit any essential part of the statement. It is worded here in homely terms in order the more to impress the pupil. The first point names the locus, the second, locates it, the third, measures it. In loci of indefinite extent, such as the perpendicular bisector of a line, there is of course no need of the third point.

The pupil in plane geometry, in its present limits, should be reminded that if it is apparent that his locus is a line that is not a straight line, it must be the arc of a circle, inasmuch as no other curve is treated in plane geometry. This does not forbid the construction by the pupil from the proper data a parabola, simply as an illustration in discovering a locus which he is pretty sure not to know. He would, however, be expected to lack the ability to name or describe this.

2. Prove that any point in the assumed locus satisfies the given condition.

For this purpose, it is usually well not to employ the tentative figure that has been used in discovering what the locus is, but to draw a new figure containing the supposed locus. Any point in this, definitely selected as a sample of them all, is then to be proved to fulfill the condition. Sometimes a single preceding proposition will cover this; at other times more or less original proof is required.

3. Prove that any point that is not in the assumed locus does not satisfy the given condition.

Sometimes it is simpler to show instead that any point that satisfies the condition lies in the assumed locus. This step, (3), is sometimes practically self-evident from step (2), and then scarcely requires proof.

The pupil should be required at first, however, to prove this step rigorously, even in the simpler cases. For example, if the question is that of the locus of a point 2 inches from a given point, he should take a point outside the circle and show that its distance from the point is greater than 2 inches, because the whole is greater than any of its parts; and similarly, with a point inside the circle.

4. Summary, showing how the locus has been proved.

This step is never absolutely necessary, but it has to the previous discussion what the final summary of a debate has to the argument, what the traditional Q. E. D. has to a demonstration, and perhaps what the Amen has to a song or prayer. It leaves us with the thought that the job is done.

C. Intersecting (or Compound) Loci.

1. The locus of points that satisfy two or more given conditions is found by constructing the locus for each and noting the intersections.

The pupil should be led to construct each of his loci quite independently of the other, merely superposing them on each other.

2. Special cases that modify part of the intersections must be particularly considered. The number of points of intersection may be zero, one, two, three, four, . . . or infinite. These special cases depend on (*a*) position, (*b*) dimension.

Too much effort can hardly be given to develop the attention and ingenuity of the pupil in detecting the peculiar situations that modify the intersections. In many cases it is adequate to let these be shown by the pupil by a drawing accompanied by a short, descriptive phrase, rather than by an elaborate statement.

3. Among the simple loci commonly met in these intersections, and with which as fundamentals the pupil should be familiar, are:

(*a*) All points at a given distance from a given point; from a given line; from a given circle.

(*b*) All points equidistant from 2 points; from 2 intersecting lines; from 2 parallel lines.

(*c*) All centers of circles of a given radius through a given point; passing through 2 given points.

(*d*) All centers of circles tangent to a given line at a given point; to a given circle at a given point; to 2 parallel lines; to 2 intersecting lines.

It is not the purpose here to give any list of locus problems. It may be suggested, however, that while the number is somewhat limited, and while firstclass locus problems are always at a premium, yet there exists a considerable quantity of ore to be worked over. The mines that exist under the topics of measurement, of proportional lines, and of areas should not be neglected. It is particularly unfortunate to attach the opening work on loci so much to points that are equidistant from certain things as to forever connect in the mind of the pupil a locus with the thought of equal distance. This is very often done through the shaping influence of the thought of the two locus problems that the pupil first meets in the traditional order, the locus of points equidistant from the ends of a line, and the locus of points equidistant from the sides of an angle.

As material for the first work on loci in plane geometry, the following propositions are especially useful:

(*a*) The locus of points equidistant from two given points.

(*b*) The locus of points at a given distance from a given point.

(*c*) The locus of points at a given distance from a given line, (1) of indefinite length, (2) of definite length.

(*d*) The locus of the mid-points of lines drawn from a point to a given line.

(*e*) The locus of the vertices of the right angles of right triangles drawn on a given hypotenuse.

(*f*) The locus of the mid-point of chords drawn in a circle from a given point on the circumference.

(*g*) The locus of points equidistant from a given point and a given line. (Not to be stated.)

More harm than good is usually done by the setting of locus problems that are nothing but puzzles, and that fail to illustrate important geometrical principles. They may serve to interest the pupil of unusual ability, but to the ordinary student they impart distaste of the whole subject.

A somewhat extraneous thought connected with the matter of loci is that the statement of a locus should seldom be used as a reason in demonstrative geometry. The reason on which a statement depends is more effective in the form of a proposition than when it is cast in the form of a locus. In certain theorems, failure to observe this suggestion leads to absolute worthlessness of proof. For example, if in the proposition that the bisectors of the angles of a triangle are concurrent, the pupil gives as a reason "the locus of points equidistant from the sides of an angle is the bisector of the angle," the application of this is exactly opposite in the later step of the proposition from what it is in the former steps, and consequently the statement can lead to nothing but slovenliness and inaccuracy on the part of the pupil.

The use of the methods of attack and procedure developed in the preceding paragraphs will aid, it is hoped, in bringing from the limbo of the mysterious a branch of geometrical work that is alike illuminating as a new outlook on its own subject matter, and valuable as an opening into the great mathematical avenues beyond into which some of our secondary school pupils are to enter.

Fred D. Aldrich.

Worcester Academy,
Worcester, Mass.